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A mathematical model of monetary circulation in mediaeval Bulgaria

Abstract

The article presents an approach to "measuring" the dynamics of monetary circulation in Bulgaria during the 13th and the 14th centuries, by constructing of appropriate functions, which present it. We consider archaeological finds of both Bulgarian and foreign coins in our country. We pay a special attention on the coins from Venice in order to study the economic influence of Venice in Bulgaria.

We use Microsoft Excel to construct the functions and their graphs. As a data source we use a comprehensive historian article on archaeological excavations of coins in Bulgaria. Many (and different) coin finds are dated to the 13th and the 14th centuries, and a few ones, dated to other periods. Therefore the historians claim that during the 13th and the 14th centuries the trade in Bulgaria was well developed. This is obvious, but using mathematical methods one can go further.

We use the principle, which historians apply. It can be formulated nearly thus: more coin excavations dated to a period, more coins in circulation were during this period. We believe our methods will support the historical research by presenting all the data about coin finds on a single graph. Thus it becomes possible to specify the periods (decades) of economic growths and those of decreases. Further the investigators of our past can combine the visual picture obtained by our method with other information and reasoning.

1. Introduction

The application of mathematical methods in historical research could enable historians to rise and test hypotheses on some dependencies or subjects, which couldn't be noticed and traced with the standard historical methods of research.

Work in this trend has been done for a couple of decades. We can mention here the attempts of mathematicians in introducing mathematical methods in historical research. A. Fomenko, S. Rachev and V. Kalashnikov [1, 2, 3, 4, 5, 6] and J. Tabov [7] applied "volume functions" in exploration of historical texts. As far as we know, this approach has not been applied to the study of numismatic collections yet. Our work presents an attempt in this direction.

On the other hand, in the field of applying mathematical methods to the study of numismatic collections we could mention Cristian Gazdac [8] and his *Apulum project*, where histograms are used to present distribution of Roman coin finds over several provinces. While he collected numerical values on the coin finds he did not consider any specific study of their chronological distribution.

We decided to study the chronological distribution of

all coin finds from Mediaeval Bulgaria, and

(II) finds of Venice coins from Mediaeval Bulgaria, both for the 13-14th centuries.

For our research we used data from the article "Monetary finds from 13-14th centuries as a source on the foreign trade of mediaeval Bulgaria" by Zdravko Plyakov [9]. Our goal was to create a model (or, more exactly, a function) of the chronological distribution of the coin finds.

In order to reach it, we firstly entered the data into Excel worksheet. On the second step, we constructed the so-called volume functions and their graphs. Finally we made some conclusions and hypotheses presented in this paper.

2. Description of the method

2.1. Extracting and organising data from the basic source

We chose the article [9], because it gives a comprehensive overview of almost all coin finds in Bulgaria in the studied period. It also includes conclusions on the economy status of our country in the past, namely that in 13th–14th century period the trade and the feudal manufacture were well developed in Bulgaria because of the many archaeological finds of mediaeval coins during and close to this period. This is the author's main and only conclusion. For the need of his research in [9], Z. Plyakov sometimes didn't specify the number of found coins and their dating of the accuracy, which is needed for our purposes. Our study requires more precise information. We need both *dating* and *number of coins* to construct our functions. Because of that reason, we extracted from [9] data only on the finds, where dating and quantity were both appropriately specified. Fortunately, the most data in [9] satisfy our requirements; the incomplete data are the minority and are randomly spread over different periods. Since we wish to obtain a rough assessment of the monetary circulation, there is no obstacle to miss these data; it does not change significantly the final results.

The data on the coins in the article [9] is grouped by the place of the respective finds. We regroup it by the ruler, who emitted the coins. We considered both Bulgarian and foreign coins, because they both were in circulation in mediaeval Bulgaria. On some foreign coins, including the Venice ones, there are stamps of Bulgarian rulers, verifying that coins' everyday use in Bulgaria was allowed. Furthermore the presence of foreign currencies is indication for foreign trade, which is both result and premise for powerful economy. The changes in the economic status of mediaeval Bulgaria is the object we explore via the changes of the monetary circulation. Hence we are interested in all the coins.

2.2. Function of Monetary Volume (FMV)

For every ordinary coin we define Individual Unit-step Function (IUF) as follows:

$$f(t)=\begin{cases}60/n, \text{ if } t\in[X_1;X_2],\\ 0, \text{ if } t\notin[X_1;X_2]\end{cases}$$

where:

$[X_1;X_2]$ is the period of governing of the ruler, who emitted this coin;

t is not a particular year but a decade;

n is the number of decades in the period $[X_1;X_2]$.

For every silver coin we define Individual Unit-step Function (IUF) analogously, replacing the number 60 with the number 480, which is 8 times bigger. The respective number for the golden ones is 6000. Thus the ratio golden coins: silver coins: ordinary coins=100:8:1.

We consider decades as smallest basic time units. Thus n is the "length" of the interval $[X_1;X_2]$. That's why periods like 1331–1343 become 1330–1340. Thus each ordinary coin has a total contribution of 60 to all decades in which it was in use i.e. all coins have equivalent contribution. For the rest two types of coins this equivalent treating also holds. The "magic" numbers 60, 480, 6000 can be interchanged with any other nonzero triple of numbers in the same ratio. But the multiples of 60 are divisible by 2, 3, 4, 5, and 6. This is convenient to be applied to periods of length 2, 3, 4, 5, and 6 decades. Thus we obtain integer values of f.

Given a ruler N. N emits k coins with IUF= $f_1(t)$ each. Define R(N)= $k.f_1(t)$. R(N) is the total contribution of the coins of N for the decade t. It's $\frac{60k}{n}$ for a decade in the interval of government and zero in the other case. Thus R(N) is similar to $f_1(t)$.

Summing up R(Z) for all the rulers from 13th–14th century period, we obtain the value of the total contribution of all the coins from 13th–14th centuries, as needed. We call the sum of all R(Z) *Function*

of Monetary Volume (FMV). The so obtained function is a mathematical model of monetary circulation.

2.3. Table creation in Excel

We first find the number of coins emitted by every ruler. Then we create in Excel a table with 40 columns, first of which corresponds to 1100–1110 decade, second – to 1110–1120 decade and so on. The explored period is 13th–14th centuries, but we consider the comprehending period 1100–1500. The last gives us opportunities to explore better some tendencies of monetary circulation at the endpoints of the period of interest. The number of rows is equal to the number of the rulers we consider. For every ruler is allocated a row in the table. We enter into the cells of the row the values of R(N) for the respective decades. Summing up the column elements, we obtain FMV. Excel performs the last operation automatically. Thus the task is done in an easy way. Consider that the archaeologists find a new quantity of coins emitted by the Venetian ruler Pietro Gradenigo, for example. We already have this ruler in our table and just add the number of these new coins to the old number of coins from him. It's another task we do easily with Excel.

2.4. Graph constructing of FMV

When the table is finished, the last row contains the values of FMV. We insert a new row above it, where we enter in the cells the decades between 1100-1500. Then we construct automatically graphs based on the values in the table. Figure 1 shows the graph we obtained, considering all the coin finds:

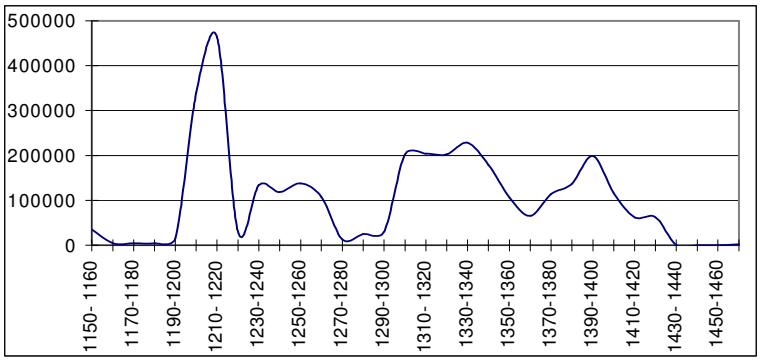

Figure 1: The graph of FMV in the interval (1150-1450).

The graph of FMV for the Venice coins is presented in Figure 2:

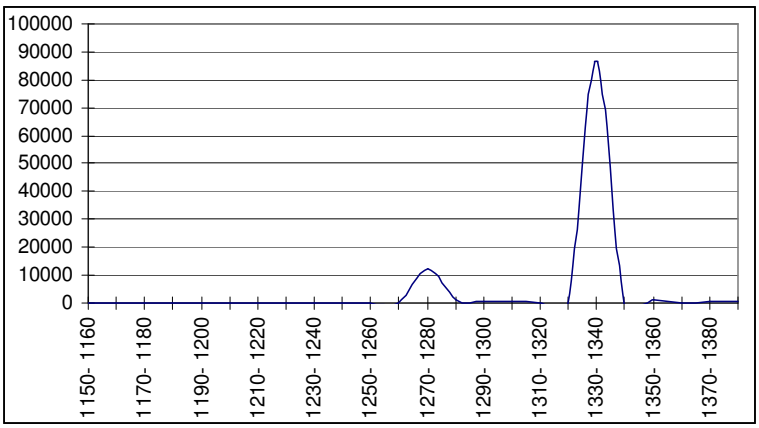

Figure 2: The graph of FMV of Venice coins from 13th–14th centuries in Bulgaria.

3. Analyses and hypotheses

Let us remind here that our source of data is the article [9]. The author's main and only conclusion there is that in 13th–14th century period the trade and the feudal manufacture were well developed in Bulgaria because of the many archaeological finds of mediaeval coins during and around this period. The same he writes about the trade relations between Bulgaria and Venice.

Our methods give opportunities to explore the dynamics of the monetary circulation. It is obvious from Figure 1 that in the period 1190-1210 the FMV has a global maximum. There are two maximums at the decades 1330-1340 and 1380-1390 as well. The first peak FMV reaches never again. The decrease after 1350, between the two maximums, is distinct and sudden. There is a serious reason for this crisis. In 1348 began a bubonic plague epidemic, which little by little spread over all South Europe. The sickness and death of numerous people caused falling in the manufactory and the trade.

The values of FMV in the interval 1260-1290 are low. During this period there was Tartar domination in Eastern Europe, including Bulgarian lands, attended with robberies, conflagrations, insecurity and panic among the people, risk for the traders etc.

Let us finally revert to the global maximum. There are some possible interpretations.

- The non-identified coins are numerous and they can change seriously the graph, if they become recognised.
- Some of the coins, dated this period, belong to a later period.

Let us now consider Figure 2. The maximum of this function at 1330-1340 coincides with a maximum of the first function. The other maximum of the Venice coins function coincides with a minimum of the first function, namely at 1270-1280. Furthermore the half of all the coins, dated to this period, are Venice. There are also Byzantine coins and a few ones coming from other countries. There is only one Bulgarian coin from this period, because of the mentioned above Tartar domination in our lands. Probably the Bulgarian economy was weak and there weren't emissions of coins. Obviously, the payments in Bulgaria were performed using foreign currencies. The most trustful currency was the Venice one.

Consider the other period (1330-1340). There are a lot of Bulgarian coins. Probably the Bulgarian economy was well developed and growing this period. Therefore we consider that the foreign coins, excavated in Bulgaria and dated this period, are due mostly to some trade relations. In that period these relations between Bulgaria and Venice were prolific, because the Venice coins are one third of all the coins dated this period. Why is that so?

4. References

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